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2749. Proposed by A. M. HARDING, University of Arkansas.

Through a point, P , within a circle, draw a chord which will be trisected at P .

2750. Proposed by J. W. LASLEY, JR., University of North Carolina.

Given $\bar{x} = \tan^{-1} \frac{z}{\sqrt{x^2 + y^2}}$, $\bar{y} = \tan^{-1} \frac{y}{x}$, and $\bar{z} = \log \sqrt{x^2 + y^2 + z^2}$, solve for x , y , and z in terms of $\bar{x}$, $\bar{y}$, and $\bar{z}$.

2751. Proposed by J. L. RILEY, Stephenville, Texas.

Every number whose square is the sum of the squares of two consecutive integers is equal to the sum of the squares of three integers of which two, at least, are consecutive.

2752. Proposed by PAUL CAPRON, U. S. Naval Academy.

Given a parallelogram with center O , vertices $PQP'Q'$, mid-points of sides $ABA'B'$ (cyclic order $PAQBPA'Q'B'$). Let K be any point of OA . Draw KLH parallel to $Q'Q$ cutting AQ at L , and draw BLM , meeting OA produced at M . Draw MH , parallel to PP' , to meet KLH at H , and draw $B'KE$ to meet BL at E . Repeat, changing A, B, P, Q to A', B', P', Q' , respectively, and *vice versa*. Repeat each of the foregoing, changing P, P', B, B' to Q, Q', B', B , respectively, and *vice versa*.

What are the loci of E and H ? Show that EH passes through A' when K is a point of OA , through A when K is a point of OA' . Consider the effect of interchanging the rôles of A and B .

(This construction, as commonly given, is specialized in these particulars: the parallelogram is rectangular, the divisions of OA are equal, and the locus of H is not found.)

SOLUTIONS OF PROBLEMS.

385 (Calculus). Proposed by H. B. PHILLIPS, Massachusetts Institute of Technology.

If $f(x)$ is continuous between a and x , show that

$$\lim_{n \rightarrow \infty} \frac{[n]}{(x-a)^n} \int_a^x \cdots \int_a^x f(x) dx^n = f(a).$$

SOLUTION BY LOUIS WEISNER, Student, College of the City of New York.

Let $f(x) = \phi^{(n)}(x)$, n being finite.

Hence,

$$\int_a^x f(x) dx = \phi^{(n-1)}(x) - \phi^{(n-1)}(a),$$

$$\int_a^x \int_a^x f(x) dx^2 = \phi^{(n-2)}(x) - \phi^{(n-2)}(a) - (x-a)\phi^{(n-1)}(a),$$

and

$$\int_a^x \int_a^x \int_a^x f(x) dx^3 = \phi^{(n-3)}(x) - \phi^{(n-3)}(a) - (x-a)\phi^{(n-2)}(a) - \frac{(x-a)^2}{2} \phi^{(n-1)}(a).$$

Integrating n times, we have

$$\int_a^x \cdots \int_a^x f(x) dx^n = \phi(x) - \phi(a) - (x-a)\phi'(a) - \frac{(x-a)^2}{2} \phi''(a) - \cdots - \frac{(x-a)^{n-1}}{[n-1]} \phi^{(n-1)}(a).$$

But

$$\begin{aligned} \phi(x) = \phi(a) + (x-a)\phi'(a) + \frac{(x-a)^2}{2} \phi''(a) + \cdots + \frac{(x-a)^{n-1}}{[n-1]} \phi^{(n-1)}(a) + \frac{(x-a)^n}{[n]} \phi^{(n)}(a) \\ + \frac{(x-a)^{n+1}}{[n+1]} \phi^{(n+1)}(a) + \frac{(x-a)^{n+2}}{[n+2]} \phi^{(n+2)}(a) + \cdots \end{aligned}$$

Hence,

$$\begin{aligned}\int_a^x \cdots \int_a^x f(x) dx^n &= \frac{(x-a)^n}{[n]} \phi^{(n)}(a) + \frac{(x-a)^{n+1}}{[n+1]} \phi^{(n+1)}(a) + \frac{(x-a)^{n+2}}{[n+2]} \phi^{(n+2)}(a) \cdots \\ &= \frac{(x-a)^n}{[n]} f(a) + \frac{(x-a)^{n+1}}{[n+1]} f'(a) + \frac{(x-a)^{n+2}}{[n+2]} f''(a) + \cdots.\end{aligned}$$

Hence,

$$\frac{[n]}{(x-a)^n} \int_a^x \cdots \int_a^x f(x) dx^n = f(a) + \frac{x-a}{n+1} f'(a) + \frac{(x-a)^2}{(n+2)(n+1)} f''(a) + \cdots$$

which when n increases without limit, becomes,

$$\lim_{n=\infty} \frac{[n]}{(x-a)^n} \int_a^x \cdots \int_a^x f(x) dx^n = f(a).$$

425 (Calculus). Proposed by O. S. ADAMS, U. S. Coast and Geodetic Survey, Washington, D. C.

Show that the infinite product

$$(1-z)(1+\frac{1}{2}z)(1-\frac{1}{3}z)(1+\frac{1}{4}z) \cdots = \frac{\sqrt{\pi}}{\Gamma(1+\frac{1}{2}z)\Gamma(\frac{1}{2}-\frac{1}{2}z)}.$$

SOLUTION BY C. F. GUMMER, Kingston, Ont.

The right member should evidently have been printed

$$\frac{\sqrt{\pi}}{\Gamma(1+\frac{1}{2}z)\Gamma(\frac{1}{2}-\frac{1}{2}z)},$$

since this equals

$$\begin{aligned}& \frac{\Gamma(\frac{1}{2})}{\Gamma(1+\frac{1}{2}z)\Gamma(\frac{1}{2}-\frac{1}{2}z)}, \\ &= \lim_{n \rightarrow \infty} \left[\left(\frac{n^{\frac{1}{2}}}{\frac{1}{2}} \prod_{i=1}^n \frac{i}{i+\frac{1}{2}} \right) \left(\frac{1+\frac{1}{2}z}{n^{1+\frac{1}{2}z}} \prod_{i=1}^n \frac{i+1+\frac{1}{2}z}{i} \right) \left(\frac{\frac{1}{2}-\frac{1}{2}z}{n^{\frac{1}{2}-\frac{1}{2}z}} \prod_{i=1}^n \frac{i+\frac{1}{2}-\frac{1}{2}z}{i} \right) \right] \\ &= \lim_{n \rightarrow \infty} \left[\frac{(1-z)(1+\frac{1}{2}z)}{n} \prod_{i=1}^n \left\{ \left(1 - \frac{z}{2i+1} \right) \left(1 + \frac{z}{2i+2} \right) \frac{i+1}{i} \right\} \right] \\ &= \lim_{n \rightarrow \infty} \left[(1-z)(1+\frac{1}{2}z) \prod_{i=1}^n \left\{ \left(1 - \frac{z}{2i+1} \right) \left(1 + \frac{z}{2i+2} \right) \right\} \right] \\ &= \prod_{i=0}^{\infty} \left\{ \left(1 - \frac{z}{2i+1} \right) \left(1 + \frac{z}{2i+2} \right) \right\},\end{aligned}$$

as required.

440 (Calculus). Proposed by J. B. REYNOLDS, Lehigh University.

If t is the differential vector joining two consecutive points on a space curve, and R is the radius of curvature at that point prove that

$$R^2 = \frac{(t \cdot t)^3}{(t \times t) \cdot (t \times t)}.$$

I. SOLUTION BY LOUIS WEISNER, Student, College of the City of New York.

Let $r = f(\tau)$ be the vector equation of the curve, s its length measured from a fixed point, and K the directed curvature.

Then,

$$K = \frac{d^2 r}{ds^2} = \frac{\frac{d}{d\tau} \left[\frac{dr}{ds} \right]}{\frac{ds}{d\tau}} = \frac{\frac{d}{d\tau} \left[\frac{t}{ds} \right]}{\frac{ds}{d\tau}},$$